First Harmonic Approximation

Power transfer deviation for resonant LLC converter

Technical Note
11 Dec 2014

Content

1. Introduction

The First Harmonic Approximation (FHA) is a modelling technique used to analyse the performance of resonant power converters [1]..[4]. When operated close to resonance, the assumption is that only the first harmonic signals contribute to power transfer. Especially for the commonly applied LLC resonant converter shown in Fig. 1 the FHA modelling technique becomes inaccurate [5]. This is mainly caused by the non-linearity of the output rectifiers.

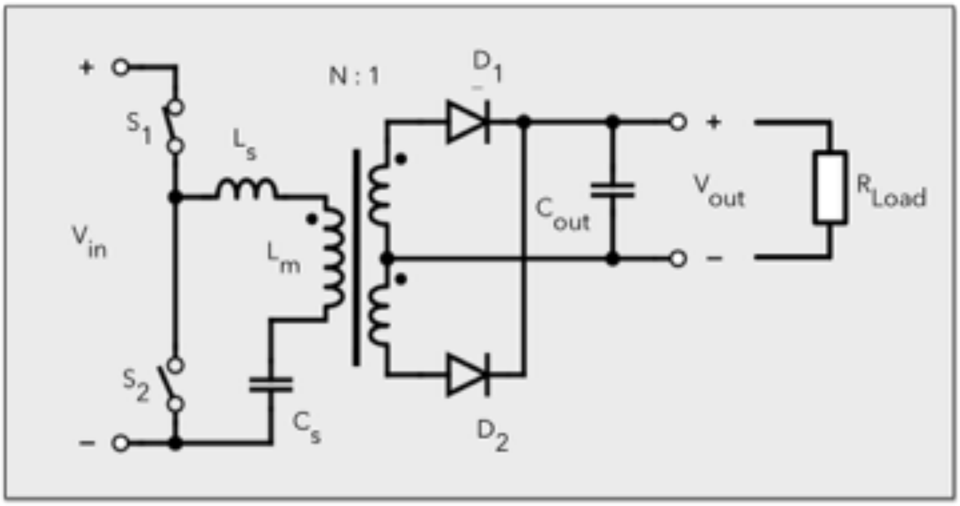

Fig. 1 Simplified LLC resonant converter diagram

This technical note will demonstrate that the power transfer estimation based upon the FHA is too conservative. A more accurate modelling technique as applied in the MATLAB tool **ZiNuZZ™** available from www.zeonpowertec.com shows the deviation between FHA and exact calculation.

2. FHA power calculation

The well known FHA equations [6] express the input voltage to output voltage transfer function as shown in (1)

$$\frac{V_{out}}{V_{in}} = \frac{1}{2 \cdot N} \cdot \frac{1}{1 + A - \frac{A}{\Omega^2} + \left(\Omega - \frac{1}{\Omega} \right) \cdot Q_e \cdot j} \qquad (1)$$

or in terms of magnitude

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{2 \cdot N} \cdot \frac{1}{\sqrt{\left(1 + A - \frac{A}{\Omega^2} \right)^2 + \left(\left(\Omega - \frac{1}{\Omega} \right) \cdot Q_e \right)^2}} \qquad (2)$$

with

$$A = \frac{L_s}{L_m} \qquad (3)$$

$$\Omega = \frac{\omega}{\omega_s}, \quad \omega_s = \frac{1}{\sqrt{L_s \cdot C_s}} \qquad (4)$$

$$Q_e = \frac{Z_s}{R_e}, \quad Z_s = \sqrt{\frac{L_s}{C_s}}, \quad R_e = \frac{8 \cdot N^2}{\pi^2} \cdot R_{Load} \qquad (5)$$

All quantities are normalised to the radial series resonance frequency ω_s. The effective quality factor Q_e relates the characteristic impedance Z_s with the effective load resistor R_e. This effective load resistor is a first harmonic representation of the actual load resistor R_{Load} taking the transformer transfer ratio into account.

Equation (2) can now be rearranged to get an explicit expression for the effective quality factor

$$Q_e = \sqrt{\frac{\left(\frac{1}{2 \cdot N \cdot \left|\frac{V_{out}}{V_{in}}\right|}\right)^2 - \left(1 + A - \frac{A}{\Omega^2}\right)^2}{\left(\Omega - \frac{1}{\Omega}\right)^2}} \tag{6}$$

The output power P_{out} can be expressed as

$$P_{out} = \frac{V_{out}^2}{R_{Load}} \tag{7}$$

and the combination with (5) and (6) yields

$$P_{out} = 8 \cdot \left(\frac{N \cdot V_{out}}{\pi}\right)^2 \cdot \frac{1}{Z_s} \cdot \sqrt{\frac{\left(\frac{V_{in}}{2 \cdot N \cdot V_{out}}\right)^2 - \left(1 + A - \frac{A}{\Omega^2}\right)^2}{\left(\Omega - \frac{1}{\Omega}\right)^2}} \tag{8}$$

Equation (8) is a FHA for the power delivered by a resonant LLC converter for a given input and output voltage (so desired gain) at a certain normalised switching frequency.

3. Actual power comparison

When compared with accurate actual calculations the FHA power estimate turns out to be too pessimistic. **ZiNuZZ**™ accurately calculates steady state output power delivery and shows the substantial divergence with the FHA power estimate.

In Fig. 2 a power converter delivering 850W at 420V input and 12V output is compared in terms of actual power (red solid line) to the FHA estimated power (red dashed line). The FHA estimate is far too pessimistic.

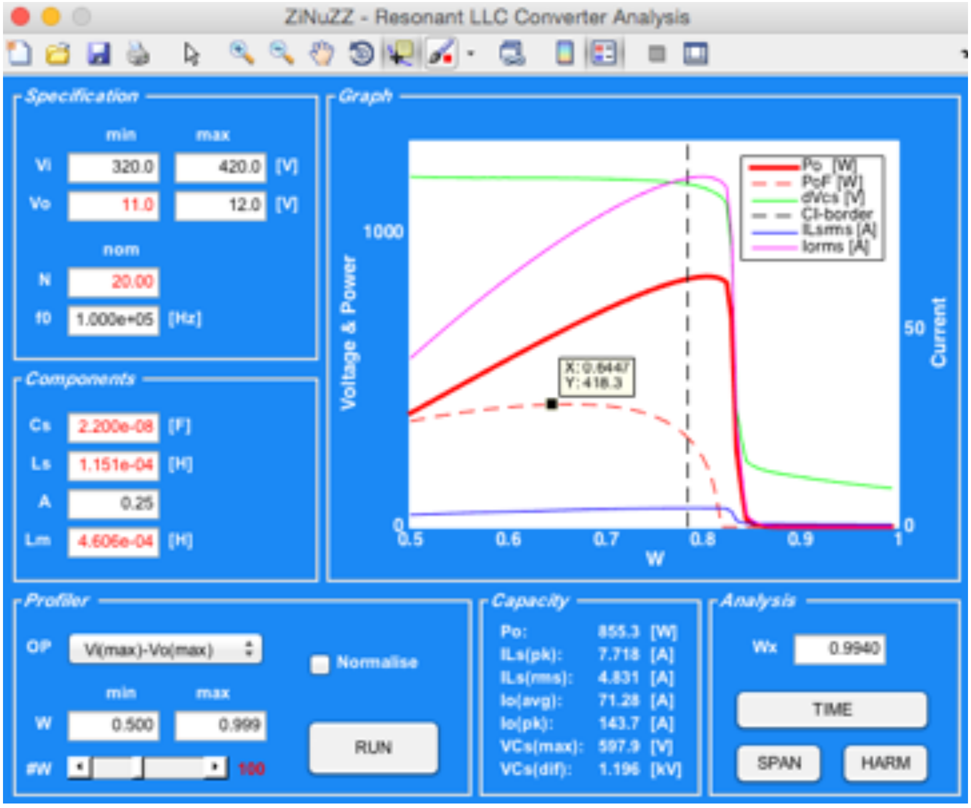

Fig. 2 12V/800W LLC resonant converter design

In a second example, shown in Fig. 3, a power converter with 390V input delivering 125W to a 24V output is shown. Again the FHA estimated power (red dashed line) is substantially more conservative than the actual delivered power (red solid line).

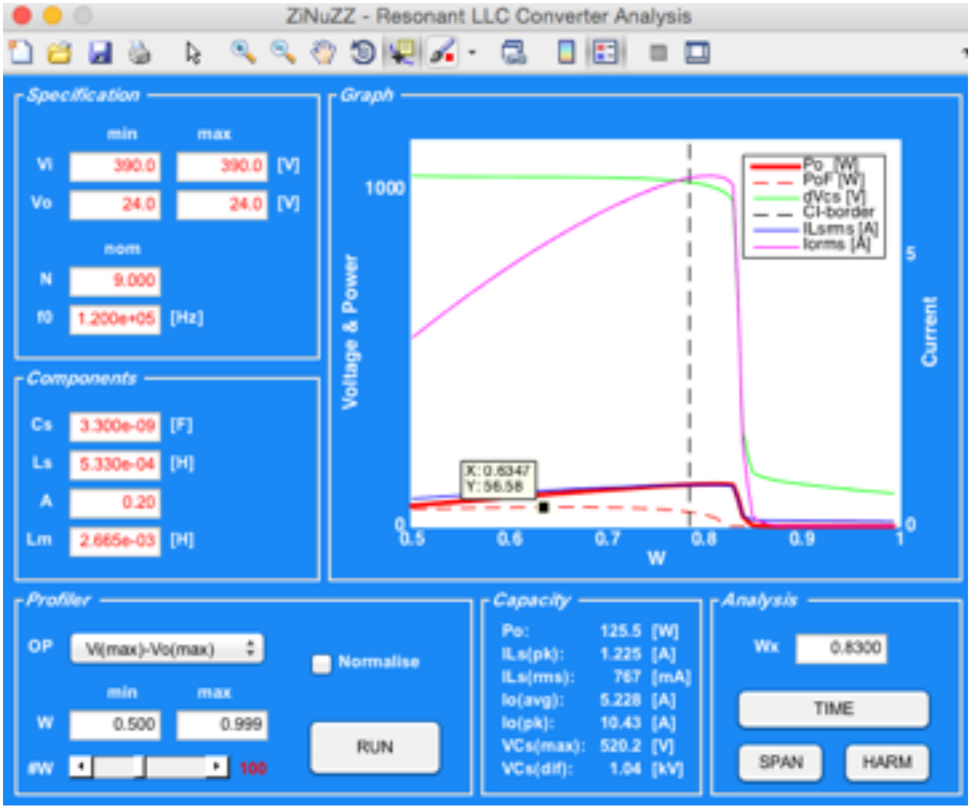

Fig. 3 24V/125W LLC resonant converter design

4. Summary and conclusions

The First Harmonic Approximation (FHA) power estimation is too pessimistic compared to the actual calculated power delivered by an LLC resonant converter. An alternative way of interpreting this observation results in the conclusion that the FHA estimated transfer gain of an LLC converter is too conservative. Numerical MATLAB tools like **ZiNuZZ™** available from www.zeonpowertec.com yield more optimised designs.

5. References

[1] Bhat, A. K. S. (1990). Analysis and design of LCL-type series resonant converter (pp. 172–178). Presented at the Telecommunications Energy Conference, 1990. INTELEC '90., 12th International. doi:10.1109/INTLEC.1990.171244

[2] Mandhana, O. P., & Hoft, R. G. (1993). Steady state frequency domain analysis of parallel and series parallel resonant converter (pp. 632–638). Presented at the Applied Power Electronics Conference and Exposition, 1993. APEC '93. Conference Proceedings 1993., Eighth Annual. doi:10.1109/APEC.1993.290734

[3] Duerbaum, T. (1998). First harmonic approximation including design constraints (pp. 321–328). Presented at the Telecommunications Energy Conference, 1998. INTELEC. Twentieth International. doi:10.1109/INTLEC.1998.793519

[4] De Simone, S., Adragna, C., Spini, C., & Gattavari, G. (2006). Design-oriented steady-state analysis of LLC resonant converters based on FHA. Power Electronics, Electrical Drives, Automation and Motion, 2006. SPEEDAM 2006. International Symposium on, 200–207. doi:10.1109/SPEEDAM.2006.1649771

[5] Oeder, C., Bucher, A., Stahl, J., & Duerbaum, T. (2010). A comparison of different design methods for the multiresonant LLC converter with capacitive output filter. Control and Modeling for Power Electronics (COMPEL), 2010 IEEE 12th Workshop on, 1–7. doi:10.1109/COMPEL.2010.5562440

[6] Huang, H. (2011). Designing an LLC Resonant Half-Bridge Power Converter. Power Supply Design Seminar (pp. 1–30). SEM1900-Topic 3. Texas Instruments.